Calculus Warm Up # 5-5

Test for convergence. State the test(s) used and give a complete conclusion.

$$\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{n+2}{n}\right)$$

<u>Euler's Method</u>: Uses local linearity to approximate a curve from a differential equation and an initial condition.

Given: $\frac{dy}{dx} = x + y$, $f(2) = 0$, Approximate f(3) using $\Delta x = 0.2$

(x, y)	$\frac{dy}{dx} = x + y$	$\Delta y = \Delta x(\frac{dy}{dx})$	y + Δy
(2, 0)	2 + 0 = 2	(0.2)(2) = 0.4	0 + 0.4
(2.2, 0.4)	2.6	(0.2)(2.6)	0.4 + 0.52
(2.4, 0.92)	3.32	(0.2)(3.32)	0.92 + 0.664
(2.6, 1.584	4.184	(0.2)(4.184)	1.584 + 0.8368
(2.8, 2.4208	5.2208	(0.2)(5.2208)	2.4208 + 1.04416
(3, 3.4649		(0.2)(	

$\boxed{f(3) \approx 3.465}$

Now try MC - A, # 7

(x, y)	$\frac{dy}{dx} = x - y - 1$	$\Delta y = \Delta x(\frac{dy}{dx})$	$y + \Delta y$
$(1,-2)$	$1+2-1 = 2$	$(0.2)(2) = 0.4$	$-2 + 0.4$
$(1.2, -1.6)$	$1.2+1.6-1 = 1.8$	$(0.2)(1.8) = 0.36$	$-1.6 + 0.36$
$(1.4, -1.24)$			

Group Quiz :

Testing for Convergence/Divergence

Writing a Taylor Polynomial for a function and using it to find an approximation.

Classwork turn in:

WU & MC-B

HW turn in:

MC-A, except # ~~7~~, 13, 17, 23, 26

HW: BC Practice FR # 4 - 6

(copy follows if you were absent) (Yellow)

AP® Calculus
Instructions for Section II Free-Response Questions

Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

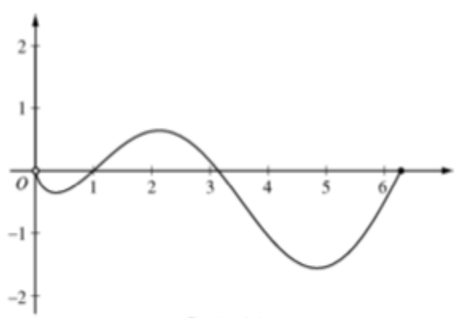
Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

   (a) Find $g(1)$ and $g'(1)$.

   (b) On what intervals, if any, is g increasing? Justify your answer.

   (c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

   (d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x\to\infty} f(x) = 2$.

(a) Find the value of $\int_0^{\infty} (4x - 2xf(x))\, dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

6. The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

(a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

(b) Let f be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

(d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.